OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS **2601**

Pure Mathematics 1

Monday **23 MAY 2005** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use only a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 4 printed pages.

[Turn over

Section A (30 marks)

1 Differentiate $x^6 - 3x^4 + 7$. [2]

2 Write $\frac{5\pi}{6}$ radians in degrees. [2]

3 Find the coefficient of x^3 in the binomial expansion of $(2 + x)^8$. [3]

4 Find, in the form $y = mx + c$, the equation of the line which passes through $(1, 4)$ and $(3, -6)$. [3]

5

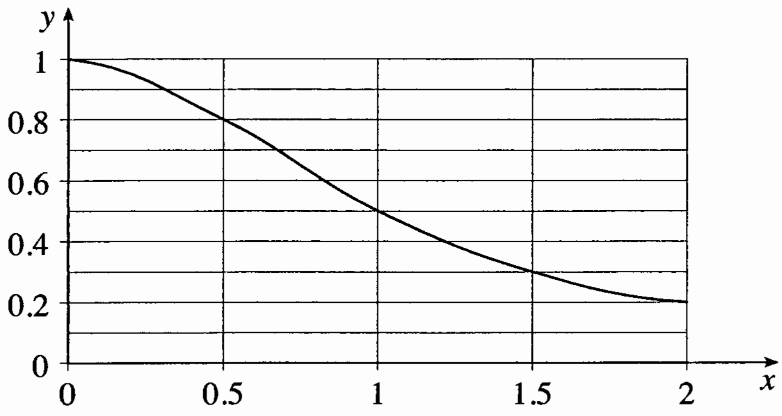


Fig. 5

Use the trapezium rule with 4 strips to estimate the area of the region bounded by the curve, the x-axis, the y-axis and the line $x = 2$ in Fig. 5. [4]

6 Solve the inequality $5x^2 - 9x - 2 > 0$. [4]

7 The angle θ is acute and $\sin\theta = \frac{3}{\sqrt{11}}$. Find the exact value of $\cos\theta$. Write down the exact value of $\operatorname{cosec}\theta$. [4]

8

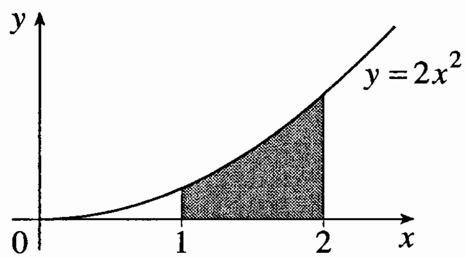


Fig. 8

The shaded region shown in Fig. 8 is rotated through 360° about the x-axis.

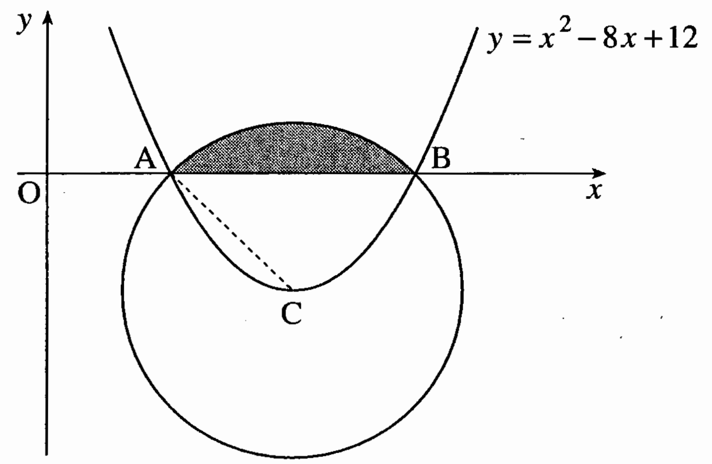
Calculate the volume which is generated. [4]

9 Three consecutive integers are written down in order. The middle integer is n. Find in terms of n the sum of the squares of the three integers. Simplify your answer.

Deduce that the sum of the squares of three consecutive integers is never divisible by 3. [4]

Section B (30 marks)

10



Not to scale

Fig. 10

The curve $y = x^2 - 8x + 12$ crosses the x-axis at A and B and has a minimum point at C as shown in Fig. 10.

(i) Show that the x-coordinate of C is 4, and find the y-coordinate. [4]

(ii) Show that the length of the straight line AC is $\sqrt{20}$.

Hence write down the equation of the circle with centre C which passes through A and B. [3]

(iii) Show that angle ACB $= 0.93$ radians to 2 significant figures.

Hence calculate the area of the minor segment of the circle which is bounded by the x-axis (shown shaded in Fig. 10). [6]

(iv) Find the area of the region bounded by the curve $y = x^2 - 8x + 12$ and the x-axis. [3]

[Turn over

11

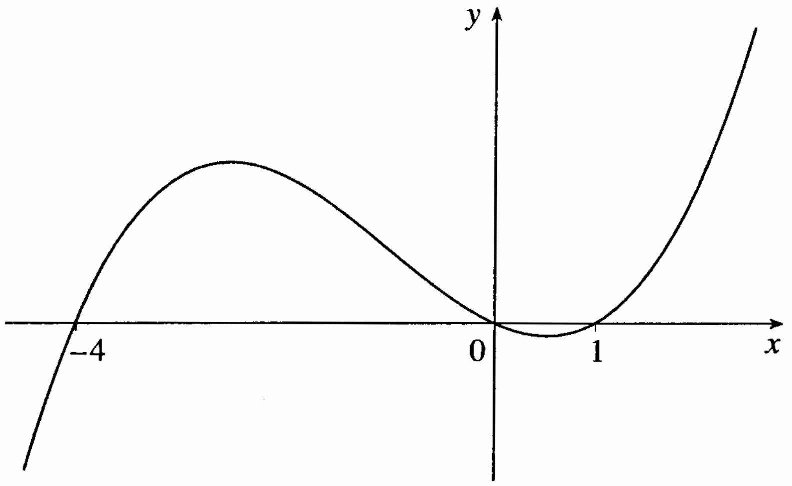


Fig. 11

The curve shown in Fig. 11 has equation $y = f(x)$, where $f(x)$ is a cubic polynomial. The coefficient of x^3 is 1.

(i) Express $f(x)$ as the product of three factors. Hence show that $y = x^3 + 3x^2 - 4x$. [2]

(ii) Use calculus to find the x-coordinates of the turning points of this curve, giving your answers to 2 decimal places. [5]

(iii) Verify that the point $P(-1, 6)$ lies on the curve.

Show that the equation of the normal to the curve at P is $y = \frac{1}{7}(x + 43)$. [4]

The normal at P cuts the curve again at Q and R.

(iv) Show that the x-coordinates of P, Q and R satisfy the equation

$$7x^3 + 21x^2 - 29x - 43 = 0.$$

Hence verify that the x-coordinates of Q and R satisfy the equation

$$7x^2 + 14x - 43 = 0.$$ [3]

Mark Scheme 2601
June 2005

	Section A			
1	$6x^5 - 12x^3$	B2	B1 if one error	2
2.	150	B2	M1 for $\times 180/\pi$	2
3.	1 792	B3	M1 for ${}^8C_3$ or 56 and M1 for 2^5	3
4.	$y = -5x + 9$	B3	M1 for gradient = $(-6 - 4) \div (3 - 1)$ and M1 for $(y - 4)$ = their $m(x - 1)$ o.e.	3
5.	at least 4 of 1, 0.8, 0.5, 0.3, 0.2 seen or used 1.1	B1 B3	M2 for $0.5/2 \times \{1 + 0.2 + 2(0.8 + 0.5 + 0.3)\}$o.e.; M1 for $k/2 \times \{1 + 0.2 + 2(0.8 + 0.5 + 0.3)\}$o.e , $k \neq 0.5$ or other single error, or for two separate correct traps [0.45, 0.325, 0.2, 0.125]	4
6.	$(5x + 1)(x - 2)$ 2 and $-1/5$ inequalities ft their roots or sketch of quadratic correct way up $x > 2$ or $x < -1/5$ or ft their roots	M1 A1 M1 A1	for attempt at factorisation or quad. formula or B2 or B4	4
7.	$\cos\theta = \dfrac{\sqrt{2}}{\sqrt{11}}$ or $\sqrt{\dfrac{2}{11}}$ cosec $\theta = \sqrt{11}/3$	B3 B1	M1 for <u>use</u> of $\sin^2\theta + \cos^2\theta = 1$ or for rt angled triangle with hyp = $\sqrt{11}$ and M1 for $\cos^2\theta = 1 - 9/11$ or $(\text{side})^2 = 2$	4
8.	integral of $\pi(2x)^2$ $[4\pi]x^5/5$ their integral at 2 – their integral at 1 $124/5\pi$ or 24.8π or 77.9(1..)	M1 M1 M1 A1	 ft for integral of $2\pi x^4$ or omission of π 0 for original fn or differential used	4
9.	$n - 1$, $n + 1$ seen $n^2 - 2n + 1$ and/or $n^2 + 2n + 1$ seen [sum of squares =] $3n^2 + 2$ e.g. '3 is a factor of $3n^2$ but not of 2' or 'there will always be a remainder of 2' o.e.	B1 B1 B1 E1		4
			Total Section A	30

		Section B			
10	(i)	$(x-2)(x-6)$ A [and B] have x coords 2 [and 6]	M1 A1	correct factors or correct use of formula; marks for A may be earned in (ii)	
		C has x coord $(2+6)/2$ [=4] C has y coord -4	M1 B1	or M1 for $y'=2x-8$ and $y'=0$ used or for $(x-4)^2-4$	4
	(ii)	$AC^2 = 2^2 + 4^2$ $(x-4)^2 + (y+4)^2 = 20$ o.e.	M1 B2	B1 for one side correct	3
	(iii)	sin (½ ACB) = $2/\sqrt{20}$ or cos (½ ACB) = $4/\sqrt{20}$ or tan = 2/4 $0.4636.. \times 2$ Area sector = $\frac{1}{2} \times (\sqrt{20})^2 \times 0.93$ Area tri. = $\frac{1}{2} \times (\sqrt{20})^2 \times \sin 0.93$ Area segment = Area sector – triangle 1.27 to 1.3	M1 M1 M1 M1 M1 A1	or M1 correct use of cos rule and M1 for cos ACB = 12/20 and completion or M1 for $8 = \frac{1}{2} \times (\sqrt{20})^2 \times \sin C$ and M1 for sin C = 0.8 and completion or angle found in degrees and conv or 9.27 to 9.3 or $\frac{1}{2} \times 4 \times 4$ or 8 to 8.02	6
	(iv)	$x^3/3 - 4x^2 + 12x$ value at 6 – value at 2 [–] 10.66 to 10.7	M1 M1 A1	condone one error dep on an integral found answer can imply 2nd M1; 0 for answer with no evidence	3
11	(i)	$y = 0$ when $x = -4, 0, 1$ so factors are $(x+4)$, x, $(x-1)$ constructive intermediate step such as $x(x^2+3x-4)$	B1 B1	marks may be earned in either order NB answer given	2
	(ii)	$y' = 3x^2 + 6x - 4$ use of $y' = 0$ attempt at subst in quad formula [x =] 0.53 or –2.53	M1 M1 M1 A2	may be earned in (iii); condone one error 1 each, or A1 for both solns not to 2 dp	5
	(iii)	$(-1)^3 + 3 \times (-1)^2 - 4 \times -1$ [= 6] subst of –1 in their y' [=–7] grad normal = –1/grad tgt [=1/7] $(y-6) = 1/7(x+1)$ cao and ft	B1 M1 M1 A1	 allow 1/7 bod if prev M1 earned and ans −7 seen there or subst of (–1, 6) in $y = 1/7\, x + c$ or given answer	4

	(iv)	$\frac{1}{7}(x+43) = x^3 + 3x^2 - 4x$ cao correct constructive step in rearranging (not just expanding bracket) $(x + 1)$ used as factor to obtain given quadratic	M1 M1 M1	 dep on previous M1 NB answer given	3

2601 - Pure Mathematics 1

General Comments

There were more candidates than expected for this legacy paper. There were a few centres who use the A level specification in a linear way and therefore entered whole groups of candidates. There were also many entries of a small number of retake candidates per centre, with candidates from across the ability spectrum using this paper in an attempt to improve their uniform score on the P1 unit.

Candidates, in general, had sufficient time to complete the paper and presented their work well.

Comments on Individual Questions

Section A

1) The differentiation was very good.

2) The conversion of radians to degrees was usually correct. A small number of candidates made errors such as inverting the conversion factor or cancelling incorrectly.

3) There was a full range of responses, from the concise ${}^8C_3 \times 2^5 = 1792$ to attempts at full algebraic expansions. The common errors were to forget the 2 or to write $2\left(1+\frac{x}{2}\right)^8$.

4) Most candidates found the equation of the line correctly. There were the usual errors such as inverting the gradient or not handling the negative correctly.

5) Candidates who handled the trapezium rule correctly were able to find the answer rather more easily than those who worked with separate trapezia or rectangles and triangles. More candidates than usual substituted x-values instead of y-values into the trapezium rule formula, often quoting 'first + last + twice the others' rather than using the given formula.

6) Most candidates were able to solve the associated quadratic equation correctly, although many resorted to the formula rather than factorising. However, the handling of the inequality was then poor, with many concluding that $(5x + 1)(x - 2) > 0$ implies $5x + 1 > 0$ or $x - 2 > 0$. The few who produced a sketch graph were usually successful.

7) The vast majority attempted to use their calculators here to find the angle or a decimal form of the fraction. Those who used the common correct methods of a right-angled triangle with $\sqrt{11}$ on the hypotenuse, or used $\sin^2\theta + \cos^2\theta = 1$, often made errors in squaring.

8) Many did not use the correct formula for finding the volume of revolution, and many of those who did made errors in squaring $2x^2$ and received only partial credit.

9) Some candidates were not able to start this question. Those who did realise that the other two numbers were $n - 1$ and $n + 1$ did not always understand 'the sum of the squares'. Many did get going and most of those were able to obtain $3n^2 + 2$ correctly, even if they then had difficulty in explaining how this result showed that the sum was not divisible by 3.

Section B

10) (i) Most candidates used differentiation to find the coordinates of C, then in the next part realised that they needed the coordinates of A. A few found A and B first by solving the quadratic equation, then used symmetry to find C.

(ii) Most candidates successfully demonstrated that the length of AC is $\sqrt{20}$. Most knew the correct form for the equation of a circle, but there were frequent errors in applying it, with some using the wrong centre and some having $\sqrt{20}$ instead of 20 for r^2, for instance. Some candidates elegantly solved these first two parts in a few lines; some took a couple of pages.

(iii) Showing angle ACB is 0.93 radians was not found easy, with many working in degrees and then converting. There were the usual errors in working with sectors and triangles; with relatively few realising they could use the simple $\frac{1}{2} \times 4 \times 4 = 8$ for the area of the triangle instead of using the angle.

(iv) This required a standard integration, with the usual errors, but many were able to complete it successfully.

11) (i) Many produced the factors immediately and usually expanded them well. A few worked backwards and factorised the given cubic.

(ii) This was generally well done, although a few candidates ignored the hint of giving answers to 2 decimal places and attempted factorisation.

(iii) The derivation of the equation of the normal was very well done, with the given answer giving candidates confidence to proceed from correctly finding $y' = -7$.

(iv) Except for the weakest candidates, many were able to equate the normal and the cubic and tidy up the resulting equation to obtain the given result. However, few realised that they needed to use the known factor of $(x + 1)$ to find the required quadratic factor.